

ANALYSIS AND COMPUTATION OF EQUILIBRIA AND REGIONS OF STABILITY

With Applications in Chemistry, Climatology,
Ecology, and Economics

RECORD OF A WORKSHOP

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MEMORANDA CIRCULATED DURING THE WORKSHOP

Rigorousness May Be Dangerous
(The "Fixed-Point Ideology")

A. Molchanov

Negative Side

Consider the equation

$$\frac{dr}{dt} = -\alpha(r, \varepsilon) \quad . \quad (1)$$

It follows from the system,

$$\frac{dx}{dt} = \partial w / \partial y - 1/2 x \alpha(r, \varepsilon)$$

$$\frac{dy}{dt} = \partial w / \partial x - 1/2 y \alpha(r, \varepsilon) \quad (2)$$

in the polar coordinate system,

$$r^2 = x^2 + y^2 \quad . \quad (3)$$

There ε is a small parameter (described more precisely below) and function $\alpha(r, \varepsilon) > 0$. (4)

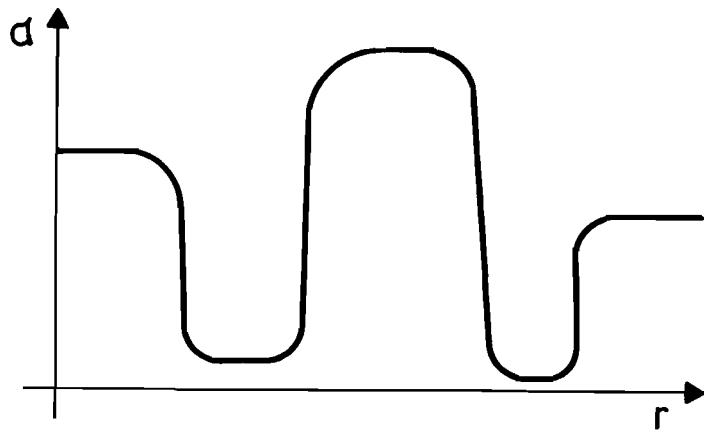
Therefore, system (2) has the Liapunov-function,

$$L(x, y) = r \quad , \quad (5)$$

and (1) may be regarded as the equation for the L - function of the system (2).

Hence, the system (2) has a unique fixed-point $x = 0, y = 0$ and no other fixed-points or limit cycles. All trajectories tend to this stable state of equilibrium.

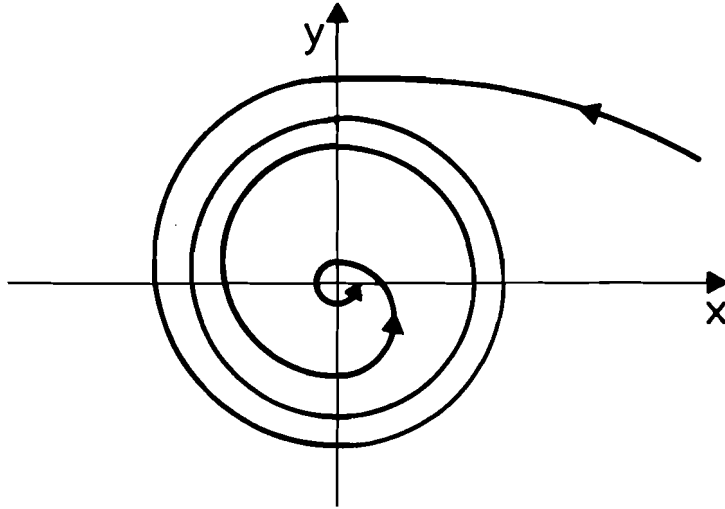
Suppose, however, that $\alpha(r, \epsilon)$ is small in some domain



For example,

$$\alpha(r, \epsilon) = \begin{cases} \epsilon & \text{if } |r-1| < \delta \\ \alpha & \text{if } |r-1| < \delta \end{cases} .$$

In this case, trajectories "drift" very slowly through the ring $|r-1| \leq \delta$.



The "through-time" T is

$$T = \int_{r_1}^{r_2} \frac{dr}{\alpha(r, \epsilon)} \approx \int_{1-\delta}^{1+\delta} \frac{dr}{\epsilon} = \frac{2\delta}{\epsilon}$$

and may be very long if ϵ is small enough.

Practically, $r = 1$ performs the role of a limit cycle. It is stable outside and unstable inside.

But rigorous mathematical considerations have omitted this important phenomenon.